Leptogenesis, Dark Energy, Dark Matter and the neutrinos

Utpal Sarkar[1]

Physical Research Laboratory, Ahmedabad - 380 009, INDIA

Abstract.
In this review we discuss how the models of neutrino masses can accommodate solutions to the problem of matter-antimatter asymmetry in the universe, dark energy or cosmological constant problem and dark matter candidates. The matter-antimatter asymmetry is explained by leptogenesis, originating from the lepton number violation associated with the neutrino masses. The dark energy problem is correlated with a mass varying neutrinos, which could originate from a pseudo-Nambu-Goldstone boson. In some radiative models of neutrino masses, there exists a Higgs doublet that does not acquire any vacuum expectation value. This field could be inert and the lightest inert particle could then be a dark matter candidate. We reviewed these scenarios in connection with models of neutrino masses with right-handed neutrinos and with triplet Higgs scalars.

INTRODUCTION

Our understanding about our universe has been improving very fast. The advancements in technology are contributing to this development tremendously. Many cosmological models have been ruled out and big-bang model of cosmology has been accepted as the base model for the evolution of the universe. The total matter content of the universe is believed to be same as the critical density of the universe. However, most of this matter is not visible, about 70 % of matter is in the form of dark energy or cosmological constant and another 25 % of matter is in the form of dark matter. Only a fraction of the remaining baryonic matter constitute the entire visible matter. This tells us that the universe is flat and accelerating at present.

Although the total amount of visible matter is very small, most of the visible matter we see around us contains almost no antimatter. This is not easy to explain. It is expected that the universe was symmetric with respect to any conserved charge. So, there would have been equal amount of baryons and antibaryons and also equal amount of leptons and leptons. Then the observed baryon asymmetry of the universe requires some explanation. Another puzzle concerns the dark matter of the universe. Which particles could be the dark matter candidates. These particles must have been frozen out at some early time and remained in the background. So, their interaction rate must be fairly small. On the other hand, if their interaction rate is too small, they might have overclosed the universe. These dark matter particle must also have contributed to the formation of the large scale structures in the universe, which requires that these particles

[1] E-mail:utpal@prl.res.in

to be non-relativistic. These cold dark matter candidates cannot be one of the known particles of the standard model. A major part of our matter content is in the form of dark energy or cosmological constant. While this is larger than all other matter contents, in particle physics standard this is extremely small. The electroweak phase transition would contribute to the dark energy of the universe, which is almost 52 orders of magnitude larger than what is required at present. Why the observed dark energy is so small requires a dynamical solution.

During the past decade many important results came out in neutrino physics. The atmospheric neutrinos and the solar neutrinos[1, 2] confirmed that the neutrinos have small masses and there are three generations of neutrinos. Including the results from laboratory experimentslike KamLAND [3] we now know the two mass-squared differences and two of three mixing angles fairly well. The third mixing angle is also constrained by the laboratory experiments. The mass-squared differences and the mixing angles are:

$$\begin{aligned} \Delta m^2_{atm} &= 2.1\times 10^{-3}\ \mathrm{eV}^2 \quad \text{with} \ \sin^2 2\theta_{atm} > 0.92 \\ \Delta m^2_{sol} &= 7.9\times 10^{-5}\ \mathrm{eV}^2 \quad \text{with} \ \tan^2\theta_{sol} = 0.4\pm 0.1\,, \end{aligned} \tag{1}$$

where $\theta_{\mu\tau}$ is the mixing angle between ν_μ and ν_τ and $\theta_{e\tau}$ is the mixing angle between ν_e and one of the other two physical states, which is an admixture of the states ν_μ and ν_τ. The absolute mass of the neutrinos have not yet been determined, although there is an upper bound on the sum over all neutrino masses from cosmology [4]: $\sum_{i=e,\mu,\tau} m_{\nu_i} \leq$ 0.69 eV. The neutrinoless double beta decay also gives an upper bound on the absolute mass of the neutrinos [5], but this bound is not valid if the neutrinos are Dirac particles.

In this review we shall point out that it is possible to provide explanation to all the three major issues in cosmology, matter-antimatter asymmetry, dark matter and dark energy, in terms of new physics coming from models of neutrino masses. While the most natural solution to the matter-antimatter asymmetry is related to the neutrino masses and called the leptogenesis, the dark matter and dark energy problems also could have an explanation coming from neutrino masses.

LEPTOGENESIS

There are several mechanisms to generate a baryon asymmetry of the universe starting from a neutral universe, but most of these mechanisms are based on one single principle proposed by Sakharov [6]. It requires three ingredients: *i*) Baryon number (B) violation, *ii*) CP violation, and *iii*) departure of the B-violating interactions from equilibrium. Instead of B violation, if the interactions violate some combinations of baryon (B) and lepton (L) numbers, an asymmetry in that quantum number will be generated. In grand unified theories all the ingredients were present [7] and this mechanism could be successfully implemented. But the asymmetry generated was a $B+L$ asymmetry and it was pointed out that this asymmetry is washed out by a new process. In the standard model, both B and L are global symmetries, but the combination $(B+L)$ is broken by quantum effects arising from anomalous triangle loop diagrams [8]. These anomaly induced $B+L$ violating processes are suppressed by the quantum tunnelling probability.

But at finite temperature, during the period $10^2 < T < 10^{12}$ GeV, these interactions become strong in the presence of some static topological field configuration called the sphalerons [9]. This will wash out the $B+L$ asymmetry generated during the GUT phase transition and make the universe baryon symmetric. A new proposal [10] was then made to make use of the sphaleron transitions to generate a baryon asymmetry from a lepton asymmetry of the universe and relate it with the neutrino masses.

The smallness of the neutrino masses requires lepton number violation at some large scale. If this lepton number violation generates a lepton asymmetry of the universe, that will get converted to the required baryon asymmetry of the universe, before the electroweak phase transition [10, 11]. Consider the most general dimension-5 effective lepton-number violating operator in the standard model [12]

$$\mathscr{L}_{Maj} = \Lambda^{-1}(\nu\phi^\circ - e\phi^+)^2, \tag{2}$$

where Λ is some lepton-number violating heavy scale and ϕ is the usual Higgs doublet scalar. The vacuum expectation value of ϕ will induce a Majorana mass to the neutrinos

$$\mathscr{L}_{Maj} = m_\nu \nu_{iL}^T\, C^{-1} \nu_{jL}, \tag{3}$$

with $m_\nu = v^2/\Lambda$. We shall now consider the two simplest realizations of this effective operator, which are the see-saw mechanism [13] and the triplet Higgs mechanism [14, 15, 16]. There are several other interesting models of neutrino masses [17]. In the left-right symmetric extensions of the standard model [18] both these contributions are natural for a particular choice of Higgs scalar [19]. In this article we shall restrict ourselves to only the seesaw and the triplet Higgs mechanisms.

To implement the see-saw mechanism [13], we introduce three right-handed neutrinos $N_{Ri}, i = e, \mu, \tau$, which are singlets under the standard model. The Lagrangian containing the Yukawa interactions of the right-handed neutrinos is given by,

$$\mathscr{L}_{int} = h_{\alpha i}\, \overline{\ell_{L\alpha}}\phi\, N_{Ri} + M_{ij}\, \overline{(N_{Ri})^c}\, N_{Rj} \tag{4}$$

where, $\ell_{L\alpha}$ are the light leptons, $h_{\alpha i}$ are the complex Yukawa couplings and α and i are the generation indices. The Majorana masses of the right-handed neutrinos M_{ij} gives the lepton number violating scale in this case and determines the mass of the light neutrinos. The Majorana mass of these right-handed neutrinos allow them to decay into a lepton and an antilepton,

$$\begin{aligned} N_{Ri} &\to \ell_{jL} + \bar{\phi}, \\ &\to {\ell_{jL}}^c + \phi. \end{aligned} \tag{5}$$

This lepton number violating decays can generate a lepton asymmetry of the universe, if there are enough CP violation, which may come from the interference of these tree-level decays with some one-loop diagrams, and these decays satisfy the out-of-equilibrium condition. This lepton asymmetry will be the same as the $B-L$ asymmetry of the universe, which is not washed out due to the sphaleron induced processes. In fact, the sphaleron induced processes will convert this lepton asymmetry into a baryon asymmetry of the universe.

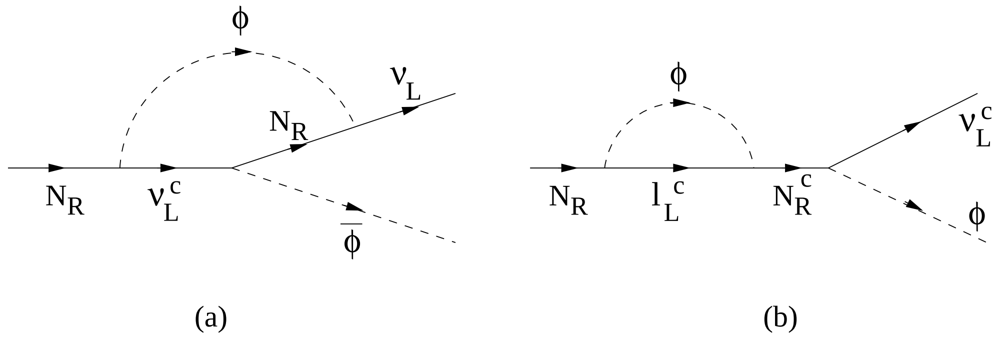

FIGURE 1. One loop (a) vertex and (b) self energy diagrams, which interferes with the tree level right-handed neutrino decays to produce CP violation.

There are two sources of CP violation in this scenario :

(i) vertex type diagrams [10, 20] which interferes with the tree level diagram given by figure 1(a). This is similar to the *CP* violation coming from the penguin diagram in $K-$decays. For baryogenesis this diagram was considered in all earlier models.

(ii) self energy diagrams [21, 22] could interfere with the tree level diagrams to produce CP violation as shown in figure 1(b). This is similar to the *CP* violation in $K-\bar{K}$ oscillation, entering in the mass matrix of the heavy Majorana neutrinos. In this case the amount of *CP* asymmetry could be very large due to resonant enhancement [22], if the masses of the right-handed neutrinos are almost degenerate.

When the decays of the right-handed neutrinos satisfies the out-of-equilibrium condition:

$$\Gamma(N \to \ell\bar{\phi}) < 1.7\sqrt{g_*}\frac{T^2}{M_P} \qquad \text{at } T = M_N, \tag{6}$$

a lepton asymmetry is generated. Here the right-hand-side correspond to the expansion rate of the universe and M_P is the Planck scale. In the case of hierarchical right-handed neutrinos, the amount of lepton asymmetry is given by

$$\begin{aligned} \delta &= \frac{\Gamma(N \to \ell\phi^\dagger) - \Gamma(N \to \ell^c\phi)}{\Gamma(N \to \ell\phi^\dagger) + \Gamma(N \to \ell^c\phi)} \\ &= -\frac{1}{8\pi}\frac{M_1}{M_2}\frac{\mathrm{Im}[\sum_\alpha(h^*_{\alpha1}h_{\alpha2})\sum_\beta(h^*_{\beta1}h_{\beta2})]}{\sum_\alpha|h_{\alpha1}|^2}. \end{aligned} \tag{7}$$

In this expression it has been assumed that the main contribution to the asymmetry comes from the lightest right handed neutrino (N_1) decay, when the other heavy neutrinos have already decayed away. The lepton asymmetry thus generated is same as the $(B-L)$ asymmetry of the universe, since there is no primordial baryon asymmetry at this time. The sphaleron interactions now convert this $(B-L)$ asymmetry to a baryon asymmetry of the universe.

We shall now discuss the triplet Higgs model [14, 15, 16]. In this case one adds two complex $SU(2)_L$ triplet higgs scalars ($\xi_a \equiv (1,3,-1); a = 1,2$). The *vev*s of the triplet

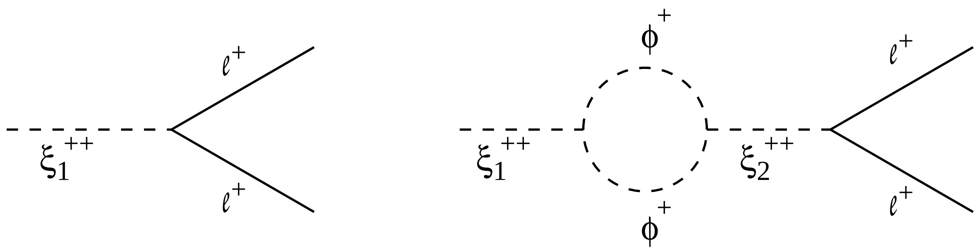

FIGURE 2. The decay of $\xi_1^{++} \to l^+ l^+$ at tree level and in one-loop order, whose interference gives *CP* violation.

higgses can give small Majorana masses to the neutrinos through the interaction

$$f_{ij}[\xi^0 \nu_i \nu_j + \xi^+ (\nu_i l_j + l_i \nu_j)/\sqrt{2} + \xi^{++} l_i l_j] + h.c. \tag{8}$$

If the triplet higgs acquires a *vev* and break lepton number spontaneously [14], then there will be Majorons, which is ruled out by the precision Z–decay width measurement at LEP. However, if lepton number is violated explicitly through a trilinear coupling of the triplet Higgs with the standard model Higgs doublet

$$V \;=\; \mu(\bar{\xi}^0 \phi^0 \phi^0 + \sqrt{2}\xi^- \phi^+ \phi^0 + \xi^{--} \phi^+ \phi^+) + h.c., \tag{9}$$

then the model is not ruled out [15, 16]. In this case a small *vev* of the triplet Higgs scalar is ensured naturally. If $\langle \phi^0 \rangle = v$ and $\langle \xi^0 \rangle = u$, then the conditions for the minimum of the potential relates the *vev* of the two scalars by

$$u \simeq \frac{-\mu v^2}{M^2},,$$

where M is the mass of the triplet higgs scalar and the neutrino mass matrix becomes $-2 f_{ij} \mu v^2 / M^2 = 2 f_{ij} u$.

The lepton number is violated at a large scale $\mu \sim M \gg v$ through the decays of the triplet Higgs scalars

$$\xi_a^{++} \to \begin{cases} l_i^+ l_j^+ & (L = -2) \\ \phi^+ \phi^+ & (L = 0) \end{cases} . \tag{10}$$

CP violation comes from the interference of the tree-level decays and the self energy diagrams of figure 2. The rate of $\xi_b \to \xi_a$ no longer remains to be the same as $\xi_b^* \to \xi_a^*$. Since by *CPT* theorem $\xi_b^* \to \xi_a^* \equiv \xi_a \to \xi_b$, it means

$$\Gamma[\xi_a \to \xi_b] \neq \Gamma[\xi_b \to \xi_a]. \tag{11}$$

This is a different kind of CP violation compared to the CP violation in models with right-handed neutrinos. The generated lepton asymmetry is given by,

$$\begin{aligned}\delta &= \frac{\Gamma(\xi \to \ell\ell) - \Gamma(\xi^\dagger \to \ell^c\ell^c)}{\Gamma(\xi \to \ell\ell) + \Gamma(\xi^\dagger \to \ell^c\ell^c)} \\ &= \frac{Im\left[\mu_1\mu_2^* \sum_{k,l} f_{1kl} f_{2kl}^*\right]}{8\pi^2(M_1^2 - M_2^2)}\left[\frac{M_1}{\Gamma_1}\right].. \qquad (12)\end{aligned}$$

Depending on the details of the model the scale of leptogenesis is determined by satisfying the out-of-equilibrium condition. The lepton asymmetry thus generated after the Higgs triplets decayed away would be the same as the $(B-L)$ asymemtry and the sphaleron transitions can then convert this asymmetry into a baryon asymmetry of the universe before the electroweak phase transition, given by

$$\frac{n_B}{s} \sim \frac{\delta}{3g_* K(\ln K)^{0.6}}.$$

Here K is the Boltzmann suppression factor. There are several possible models where the required baryon asymmetry of the universe could be achieved.

DARK ENERGY

Although most of the matter density in the universe is in the form of dark energy or cosmological constant, the mass scale corresponding to the present value of the cosmological constant is only comparable to the neutrino mass scale. On the other hand, the electroweak symmetry breaking can contribute about 52 orders of magnitude larger value for the dark energy. It is possible to fine-tune some parameters to cancel the large contribution of the electroweak symmetry breaking, but such fine-tinung has to be performed to all orders of perturbation theory, which is unnatural. Many attempts to solve the cosmological constant problem has been made, but the most popular one is the quintessence solution [23]. There are different quintessence models, which differ in their predictions for the equation of state of the dark energy, $\omega = p/\rho$, where p and ρ are the pressure and the density of the dark energy. But the present value [4] of $\omega = -0.98 \pm 0.12$, supports the simplest model with $\omega = -1$.

The quintessence solutions [23] of the cosmological constant problem assumes that the mass density of a scalar field ϕ gives the dark energy. If this field has certain typical potential that allows slow variation of the field, the dark energy decreases with time. The scalar field could be the inflaton field that drives the inflation or the cosmon field associated with the the dilatation symmetry [23], or pseudo Nambu-Goldstone bosons that is presently relaxing in its vacuum state [24] or some other scalar field depending on the model.

In recent times the coincidence between the neutrino mass scales and the scale for cosmological constant has been exploited to propose a neutrino mass varying solution to the dark energy problem [25]. The neutrino mass m_ν is assumed to be a function of a scalar field, the acceleron $\mathscr{A}$, which drives the universe to its present accelerating phase.

In the non-relativistic limit, the thermal background of the neutrinos and antineutrinos with a number density n_ν gives a contribution to the potential amounting to $m_\nu(\mathscr{A})n_\nu$. There is also the vacuum energy $V_0(m_\nu)$ attributed to the acceleron $\mathscr{A}$, so that the effective potential reads

$$V(m_\mu) = m_\nu n_\nu + V_0(m_\nu)\,. \tag{13}$$

The relationship of $\omega(t)$ with the scalar potential is given by

$$\omega + 1 = -\frac{m_\nu\, V_0'(m_\nu)}{V} = \frac{\Omega_\nu}{\Omega_\nu + \Omega_{DE}}. \tag{14}$$

The observed value of $\omega \approx -1$ implies that the energy density in neutrinos is much smaller than the total energy density, which corresponds to $m_\nu\, V_0' \ll V(m_\nu)$ and hence a flat potential, which could be taken to be

$$V_0(m_\nu) = \Lambda^4 \log\left(1 + \left|\frac{M_1(\mathscr{A})}{\mu}\right|\right). \tag{15}$$

The form of $M_1(\mathscr{A})$ determines the dynamics of the model.

We shall now discuss a possible source of the acceleron field, which could be a pseudo-Nambu-Goldstone boson (pNGB) originating from lepton flavor violation [26]. Consider a a two-generation example with two right-handed neutrinos $N_i, i = 1,2$, whose Yukawa couplings with some scalar singlet fields $\Phi_i, i = 1,2$ is given by

$$Ł_N = \frac{1}{2}\alpha_1 \bar{N}_1 N_1^c \Phi_1 + \frac{1}{2}\alpha_2 \bar{N}_2 N_2^c \Phi_2. \tag{16}$$

The transformation of these fields under a global $U(1)_1 \times U(1)_2$ symmetry of the model can be read as $N_1 \equiv (1,0)$, $N_2 \equiv (0,1)$, $\Phi_1 \equiv (2,0)$ and $\Phi_2 \equiv (0,2)$ respectively. The vacuum expectation values of the fields Φ_i break both these symmetries leading to two Nambu-Goldstone bosons. One of these Nambu-Goldstone bosons remain massless, which corresponds to total lepton number, which is the singlet Majoron. The other Nambu-Goldstone boson eventually picks up a finite and small mass and become pseudo-Nambu-Goldstone boson (pNGB).

We further assume that the left-handed doublets $\ell_{\alpha L}^T \equiv (\,\nu_\alpha \quad \alpha\,)\,, \alpha = e, \mu$ transform under $U(1)_1 \times U(1)_2$ as (1,0) and (0,1), respectively. If the standard model Higgs doublet is allowed to have the usual Dirac Yukawa couplings with the neutrinos, then that will break the global symmetry explicitly. However, this will spoil the renormalizability of the theory and the pNGB will receive infinite corrections to its mass and will have a mass of the order of $\langle\Phi\rangle$. So, all Yukawa couplings should remain invariant under the global $U(1)_1 \times U(1)_2$ symmetry, which could be achieved by introducing three Higgs doublet fields $H_a, a = 0,1,2$, which transforms under $U(1)_1 \times U(1)_2$ symmetry as $(0,0), (1,-1)$ and $(-1,1)$, respectively. The Yukawa couplings

$$\mathscr{L}_{mass} \;=\; f_{11}\bar{N}_1\ell_1 H_0 + f_{12}\bar{N}_1\ell_2 H_1 + f_{21}\bar{N}_2\ell_1 H_2 + f_{22}\bar{N}_2\ell_2 H_0 \tag{17}$$

will then give the Dirac masses for the neutrinos

$$Ł_{mass} = m_{ij}\bar{N}_i \nu_j,$$

which breaks the $U(1)_1 \times U(1)_2$ symmetry softly. This will then give a finite small mass to the pNGB, which can be evaluated from the one-loop Coleman-Weinberg potential. This light pNGB with mass of the order of the neutrino masses $m_{\mathscr{A}} \sim m^2/M$ could then become the acceleron field.

After proper transformations and taking care of the redundant phases, we can write down the neutrino masses as

$$\mathscr{L}_\mu = \frac{1}{2} M_i(\mathscr{A}) \bar{N_i^c} N_i + m_{ij} \bar{N_i} \nu_j + H.c. \tag{18}$$

The right-handed neutrino mass shows explicit dependence on the acceleron field and $M_i(\mathscr{A})$ is specified. Then the effective neutrino mass also varies $m_\nu(\mathscr{A}) = m^T M^{-1}(\mathscr{A}) m$, and we finally arrive at

$$-\mathscr{L}_{eff} = m_{ij}^T [M_i(\mathscr{A})]^{-1} m_{ij} \; \nu_i \nu_j + H.c. + \Lambda^4 \log(1 + |M_1(\mathscr{A})/\mu|), \tag{19}$$

whose minima corresponds to $\omega = -1$.

The models of mass varying neutrinos have some more problems [27], which are being taken care of in some variants of the model [28, 29]. The naturalness problem could be softened drastically in models with triplet Higgs scalars [28], which provides interesting phenomenology at LHC. Like any models of quintessence, the models of mass varying neutrinos are not yet fully consistent. However, the rich phenomenological implication of this model makes it interesting. One of the implications is the existence of a long range force that could be tested in the near future.

DARK MATTER

There are many dark matter candidates, including the lightest supersymmetric particle, which is usually a neutralino. Here we shall restrict ourselves to the dark matter candidate that appears from models of neutrino masses [30]. One natural explanation of the smallness of neutrino masses is due to radiative models. In one such model one introduces an additional Higgs doublet η. An extra Z_2 symmetry prevents this Higgs from coupling with the quarks. The right-handed neutrinos (N_i, i = 1,2,3) are also odd under the Z_2 symmetry so that the scalar potential for the fields ϕ and η and the Yukawa interactions of the right-handed neutrinos are given by

$$V = \mu_1^2 |\phi|^2 + \mu_2^2 |\eta|^2 + \lambda_1 |\phi|^4 + \lambda_2 |\eta|^4 + \lambda_3 |\phi\eta|^2 + \lambda_4 |\phi^\dagger \eta|^2 + \frac{\lambda_5}{2} [(\phi^\dagger \eta)^2 + H.c.]. \tag{20}$$

$$\mathscr{L}_Y = h_{\alpha i} \, (\nu_\alpha \eta^\circ - \ell_\alpha \eta^+) N_i + H.c. + M_i N_i N_i \tag{21}$$

We now assume that only the standard model Higgs ϕ acquires a *vev*, $\langle \phi \rangle = v$ and $\langle \eta \rangle = 0$. The fields η^+ and η_I° will become the longitudinal modes of the gauge bosons making them massive and the usual physical Higgs scalar ϕ_R° can now become as heavy as 400-600 GeV. The scalars η^+, η_R° and η_I° do not interact with the light fermions and remain inert. It is now possible to choose the coupling constants of the scalar potential,

which will make one of the neutral components of η to be the lightest inert particle (LIP). The Z_2 symmetry will make the LIP to be a stable particle and hence a candidate for the dark matter of the universe. The parameters of the scalar potential can be constrained by the present limits on the dark matter. The strongest requirement is $\lambda_5 \neq 0$, which is also required for the neutrino masses. The neutrino mass matrix is then given by

$$\mathscr{M}_{\nu\alpha\beta} = \sum_i \frac{h_{\alpha i} h_{\beta i} M_i}{16\pi^2} \left[\frac{m_R^2}{m_R^2 - M_i^2} \ln\frac{m_R^2}{M_i^2} - \frac{m_I^2}{m_I^2 - M_i^2} \ln\frac{m_I^2}{M_i^2} \right] . \qquad (22)$$

We assumed the Majorana masses of the right-handed neutrinos are real and diagonal. The lightest inert particle can now have a mass in the range of 50-100 GeV and can be a candidate for the cold dark matter in the universe.

SUMMARY

Our present understanding of our universe suggests that about 70% of matter is in the form of dark energy, about 25% is non-baryonic nonrelativistic dark matter and a fraction of the remaining matter is visible matter and there is very little scope of antimatter. Considering that the total matter is about the critical matter density of the universe, this points towards the neutrino mass scale for all the matter. There are now suggestions to explain why there are more matter compared to antimatter, what are the dark matter and how to explain why the dark energy is so small, all in terms of some properties of neutrino physics. We discussed these solutions in a couple of models of neutrino masses, namely the one with right-handed neutrinos and the triplet Higgs scalars.

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